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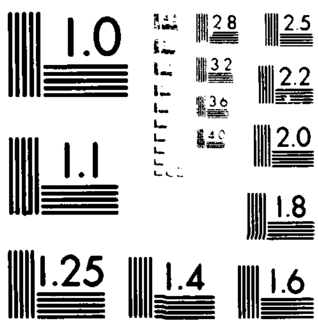

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THE MOBILITY OF WATER IN FROZEN SOILS (U)

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INTRODUCTION

The movement of water in soil systems has long been of interest for applications such as road, dam, and reservoir design; agronomy; and ground thermal storage. Recent proposals for buried, chilled pipelines raise the possibility of water movement in freezing soils over very long periods of time. The need to apply soil water and ice relations to practical problems such as frost heave and thaw weakening has prompted an expansion of research, since rules of thumb have repeatedly failed. The physics of the processes must be studied in increasing depth to rationalize mathematical models.

The relation of theoretical microscopic soil water physics to observed macroscopic parameters has been evaluated here, emphasizing frost heave phenomena. Experimental work was carried out to determine how water moves in isothermal frozen clays and to measure simultaneous pressures and temperatures within the freezing zone of silts.

SOIL WATER AND ICE

Taber [1] showed that the major cause of volume increase when soil freezes is the formation of segregated ice: ice formed from water in excess of that originally in the soil. A complete, quantitative explanation for the movement of excess soil water in response to a freezing condition is still sought. Beskow [2] demonstrated that a pressure gradient, or a gradient of free energy, developed within the water of fine-grained soil during freezing. Soil water will move under appropriate potential gradients and an understanding of these fluxes in frozen soils is vital to explain frost heaving.

Most descriptions view the soil as a macroscopic continuum with reference made to microscopic phenomena at the length scale of a single pore. Thus basic laws are cast in terms of local averages which allow equations to be written in continuous variables despite the microscopic discontinuities. Local microscopic mechanical and thermodynamical equilibria are usually assumed. Macroscopic nonequilibria, such as temperature variation, may be viewed as driving potentials, while equilibrium relations are applied locally. This approach is valid in soil physics since events occur slowly and phases are in contact over large specific surfaces. Soil hysteresis can greatly complicate computations, but no frost heave theories as yet include it. Latent heat effects are large and may be computationally troublesome, especially as they enter via non-linearities; references [3, 4] give useful reviews. If phase transition is concentrated within a macroscopic discontinuity, then a variety of methods recently devised for Stefan-type problems may be used [5-7]. If the scale of analysis entails a spatially extended phase change zone, a non-step freezing zone representation follows naturally [8, 9].

Conservation principles of momentum, energy, and mass enter into a complete frost heave model. Experimental evidence shows that an increase in overburden pressure diminishes the heave produced by forces acting in the freezing zone [10]; overburden pressure is defined macroscopically as the net force per unit area over all phases and constituents. The pressure in any phase will normally differ from the overburden pressure, but may equal it in special circumstances. If the pore water gage pressure in a saturated, coarse-grained, unfrozen soil is zero, then a weight upon the soil must be supported entirely by an "effective stress" transmitted through mineral intergranular contacts. Microscopically the effective stresses are complex and vary abruptly, but macroscopically they manifest themselves simply as a net intergranular force per unit area of soil.

The overburden pressure is not balanced by an effective stress reaction alone but also by the pore content stress. The macroscopic partitioning of pore stress, or "neutral stress," by the pore constituents results from a microscopic interplay of constituent stresses, interfacial energies, and geometry. Miller's [11] suggestion that the use of a stress partitioning factor to quantify the relative participation of macroscopic pore constituent stresses is experimentally supported by [12]. Using thermodynamics, equations were derived relating overburden pressure, ice pressure, liquid water pressure and volume ratios of ice and liquid [13, 14]. Agreement, however, has not yet been reached on the quantitative distribution of stress within the phase change regions.

One may question the validity of viewing stresses as having pressure-like manifestations macroscopically, especially when the stresses refer to an ensemble of grains, or act within extremely thin adsorbed water layers. It has been suggested that film water can pull nearby water into it while

simultaneously pushing away the ice, if the water is stressed anisotropically [15, 16]. However, explanations of the surface layer forces which separate a particle from nearby ice can be advanced without recourse to peculiar water or stresses [17-19]. Data on the film water may not be necessary, if it is locally in equilibrium with the experimentally accessible pore water. Osmotic pressure also enters into the film water - pore water - ice pressure relationships [20]. However, no frost heave model takes account of the effects of solutes nor the effects due to ions in clays.

Ice can maintain a complex state of stress and develop stress concentrations, although it tends to "relax" and relieve such conditions. The idea of simple ice pressure assumes that events happen slowly, allowing micro-variations of stress to be relieved. Since the unfrozen water attains a locally uniform state and the water coats the pore ice which forms in equilibrium with it, the ice will also tend to uniformity. When the soil matrix and unfrozen water react to an imposed load, they may move in any direction, equalizing the apparent state of stress in all constituents. The pore ice may also be considered to be isotropically stressed, behaving as a liquid-like component, with any macroscopic anisotropy entering through the stress-strain characteristics of the granular skeleton.

PHASE COMPOSITION

The water in a soil does not all freeze at a unique temperature; rather, the unfrozen water content decreases with temperature [21], and some temperature drop below 0°C is required before pore ice can exist. The finer grained a soil is, the greater the freezing point depression, and the more unfrozen water it will retain under a given set of conditions. Williams and Burt [22] measured the hydraulic conductivity of frozen soil using a permeameter with an aqueous solution of lactose. However, the lactose complicates the interpretation of the data. To overcome this difficulty Horiguchi and Miller [23] measured the hydraulic conductivity of frozen soil in the temperature range 0°C to -0.15°C using pure, supercooled water. These experiments quantify the moisture flux in frozen soils.

The partially frozen nature of heaving soil accounts for the important unfrozen water flow occurring within the warmest, least frozen portion of the frozen soil. Evidence shows ice segregation occurring within the frozen zone, behind a "frozen fringe" [24, 25]. This fringe may be defined as the zone over which volumetric ice content increases from zero at the freezing front (0°C isotherm) to 100% at the warmest lens. High overburden pressures and low temperature gradients tend to increase the fringe thickness.

An equilibrium relation which appears applicable to the ice-liquid mixture in frozen soil is the generalized Clapeyron equation [26]:

$$(p_w-\pi)v_w - p_i v_i = \frac{L(T-T_o)}{T_o} \quad , \qquad (1)$$

Applied microscopically, this equation can be used to explain equilibrium between pore ice and unfrozen water. Macroscopically, p_w and π in this equation presumably pertain to measured pore water values. The Clapeyron equation has been linked with another relation which is usually presented in terms of soil capillarity. When different water phases co-exist with curved phase interfaces (as in soil pores), there is a pressure jump between the two phases due to interfacial effects,

$$p_j - p_w = \sigma_{jw} F(\theta) \qquad (2)$$

where j refers to ice in saturated, frozen soil, or air in unsaturated, unfrozen soil. $F(\theta)$ is a macroscopic function of unfrozen water content, which corresponds to the mean curvature of the microscopic phase interface. Koopmans and Miller [27] hypothesized that $F(\theta)$, at a fixed water content, was the same for an unsaturated, unfrozen soil and a saturated, frozen soil. If p_w is zero, then from Eq. (2)

$$p_i/p_a = \sigma_{iw}/\sigma_{aw} \qquad (3)$$

For the same granular soil, Koopmans and Miller [27] found the pressure ratio to be about 0.45, indicating that capillary effects dominate in the same way for both freezing and drying. For a clay the ratio was unity.

When a particular unfrozen, unsaturated soil is considered, the curves of Eq. (2) are often called soil moisture characteristic curves. Typically, the curves differ somewhat for drying or wetting processes (hysteresis). At atmospheric air pressure, neglecting hysteresis, one obtains a curve relating moisture content and liquid pressure. With a frozen, saturated soil, one may construct freezing characteristic curves by relating θ to p_w and T using Eqs. (1) and (2). For a fixed value of p_w, a relation is found between θ and T. Thus unfrozen water relations, discussed earlier, are special cases of freezing characteristic curves at constant pressure. One cannot obtain liquid pressure in a freezing soil directly from an unfrozen moisture characteristic curve. $F(\theta)$ may be the same in both cases, but the ice-water surface tension σ_{iw} must be used in place of σ_{aw} and the product $\sigma_{iw} F(\theta)$ only provides values for the difference between ice and liquid pressures. The ice pressure need not be zero when the overburden pressure is small because, in general, any number of combin-

ations of liquid and ice pressure may hold for given values of θ and overburden pressure. Data do not support application of the same characteristic curve data to both frozen and unfrozen soils, if both soils are unsaturated [20]. In the freezing of unsaturated soil, three pore phases are present, and a more complex relation between pore contents must hold than that in Eq. (2); nevertheless, characteristic curves from unfrozen, unsaturated soil are generally used in frost heave models of unsaturated, freezing soil.

The movement of moisture in soil systems is usually described by some form of Darcy's law. This expresses the conservation of momentum when inertial effects are negligible. The gradient of free energy, or thermodynamic potential, represents a net force balanced by the drag of other components on the liquid, which is proportional to the flow rate. Thus the moisture flux can be written

$$\bar{q} = -k \nabla f \tag{4}$$

The difference in free energy of the soil moisture is the work required to change the water from one state to another. This gives the clearest physical picture of the driving potential for soil water movement since it contains all of the component potentials usually encountered, e.g. pressure, gravity electrostatic forces, osmosis. The free energy can be expressed as a function of external force fields and contact forces. Thus it is common to write the motive potential directly in terms of pressure and gravity fields:

$$\bar{q} = -k \nabla \left(\frac{\psi}{\rho_w g} + z\right) \tag{5}$$

The soil suction ψ is a combination of hydrostatic pressure, surface tension and adsorption forces of the soil skeleton. The soil suction ψ can be expressed as a function of water content and temperature, as noted already. Then the extended Darcy's law is

$$\frac{\bar{q}}{\rho_w} = -D_\theta \nabla \theta - D_T \nabla T - k \bar{e}_z \tag{6}$$

Eq. (6) may be substituted into the conservation of mass equation to arrive at

$$\frac{\partial \theta}{\partial t} = \nabla [D_\theta \nabla \theta + D_T \nabla T + k \bar{e}_z] \tag{7}$$

No frost heave models have used the mass flux relation as a function of both pressure and temperature gradients, exemplified by Eq. (6). If one-dimensional flow is considered, with negligible temperature effects, then

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(D\,\frac{\partial\theta}{\partial x}\right) \tag{8}$$

In a number of frost heave models, soil moisture diffusivity is used in the same manner for frozen and unfrozen soil, evidently by analogy between drying of unsaturated soil and freezing of saturated soil. However, the analogy may not hold. In fact, the D_T term can be dominant in saturated, granular soil. Unlike p_a, the ice pressure will not be constant in space, but should vary from some small value where ice content is low to the overburden pressure where segregation is occurring. According to Eq. (2), the liquid pressure gradient is then

$$\frac{\partial p_w}{\partial x} = \frac{\partial p_i}{\partial x} - \sigma_{iw}\left[\left(\frac{\partial F}{\partial\theta}\right)_T \frac{\partial\theta}{\partial x} + \left(\frac{\partial F}{\partial T}\right)_\theta \frac{\partial T}{\partial x}\right] \tag{9}$$

Thus the liquid pressure gradient depends upon ice pressure gradient, and gradients in θ and T. No simple relation for diffusivity exists unless the ice pressure and thermal gradients are negligible.

Pressure or temperature gradients may cause relative motion between ice and small particles completely enveloped in the ice [18, 28]. The processes involved are not completely understood but the ice must melt on one side of a particle, with liquid being transported around the particle to a location where it refreezes. The process, called regelation, may operate in heaving soils, so that movement of ice as well as unfrozen water may occur [11]. The extent of such activity in soil has not been established, but the possibility of pore ice movement exists.

ISOTHERMAL TRANSPORT OF WATER IN SUBFREEZING SOIL WITHOUT ICE

Frozen soils need not contain ice if the moisture content is less than the equilibrium value for the temperature of interest. To examine the movement of water under such conditions, tests were run whereby wet and dry soil columns were joined and the water was observed to move from the wet, ice-free part of the frozen soil column to the dry region [29].

Assuming the extended Darcy law is valid, the movement of the soil water is governed by Eq. (8), with the following conditions:

$$\theta(x,0) = \begin{cases} \theta_o & x < 0 \\ \theta_c & x \geq 0 \end{cases} \tag{8a}$$

$$\left.\begin{aligned} \lim_{x\to-\infty} \theta(x,t) &= \theta_o \\ \lim_{x\to+\infty} \theta(x,t) &= \theta_c \end{aligned}\right\} \quad t > 0 \qquad (8b)$$

Nakano [30] showed that a weak solution of Eq. (8-8b) converges asymptotically to a similarity solution of Eq. (8), assuming that the soil diffusivity is given by

$$D(\theta) = D_o\,(\theta-\theta_c)^\beta \qquad (10)$$

An experimental asymptote can be obtained at some finite time, chosen so that the moisture profile no longer varies with respect to a similarity parameter. With a best fit of the experimental and theoretical profiles the values of D_o and β can be found. This method is also applicable to the determination of the diffusivity for unfrozen porous media and is a useful new technique.

The soil was Morin clay with a specific surface area of 60 m^2/g-soil. The measured unfrozen water content (with 21% total water content) versus temperature showed that about 10% of the water remained unfrozen at -1.0°C.

Graduated plastic pipettes were used to enclose soil columns 0.80 cm in diameter by 20.33 cm in length. Two soil columns with the same soil dry density of 1.3 g/cm^3 were required per test, one uniformly dry and the other uniformly wet at 8.0%. The thoroughly mixed water and soil was allowed to set for at least three days to attain moisture uniformity.

An experiment began by joining pipettes with dry and wet soil, both held at -1.0°C. The surfaces of contact were smoothed to avoid contact resistance. The joined pipettes were then placed in a constant temperature bath containing an ethylene glycol/water mixture held at -1.0°C and maintained to within ±0.03°C. After a specified time, the compound pipette was removed from the constant temperature bath and quickly sectioned into 64 equal segments for water content determination.

Tests were carried out with time durations of 2, 4, 8 and 16 days and also at 4, 8, 16 and 32 days, to examine the reproducibility of the data. The water content is plotted versus the distance in Fig. 1. The two sets of profiles, after 8 days, indicate the difficulty in reproducing the same profile. There was about 0.5% water content difference in the wetter end of a soil column. The water content θ is plotted versus a similarity parameter ξ (Fig. 2), which indicates that the experimental profiles tend to stabilize as if they were converging upon an asymptotic profile reasonably well after 8 days. Using the least-squares method the parameters in Eq. (10) were determined as

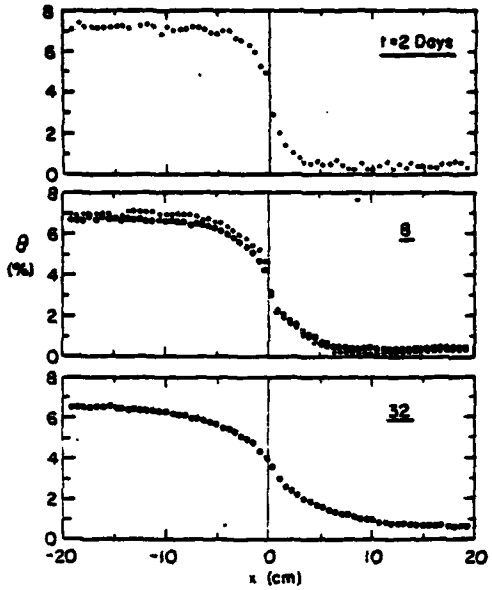

Figure 1. Water movement in ice-free Morin clay. θ_o = 8.0%; T = -1.0°C.

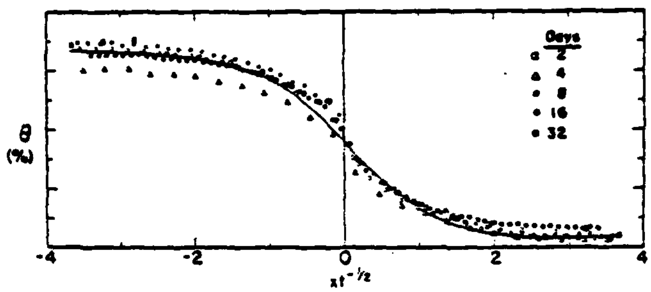

Figure 2. Similarity correlation of water content in ice-free Morin clay. θ_o = 8.0%; T = -1.0°C.

$$D(\theta) = 1.21\ (\theta - .0038)^{.272} \qquad (11)$$

where D(θ) is in cm²/day and θ is in g water/g dry soil. The curve in Fig. 2 is the computed θ profile using Eq. (11) and is a reasonably good approximation, but is not quite a close fit. If D(θ) is evaluated directly from the experimental profile at 8 days, using the same mathematical basis, it appears that there is a slight peak in D(θ) near θ = 1%. The validity of this calculation is uncertain, due to the error in the numerical evaluation of derivatives, but it is possible that the fit is somewhat erroneous because the actual D(θ) may not be a monotonically increasing function of θ.

ISOTHERMAL TRANSPORT OF WATER IN SOIL WITH ICE

Tests were run using the same technique and apparatus as described in the previous section except that the water content was adjusted so that ice was present in the wet section [31]. The total water content was 20.0% and the dry density of the clay was 1.42 Mg/m³. The unfrozen water content was evaluated by a pulsed nuclear magnetic resonance (NMR) technique to yield the following unfrozen water relation for a warming soil:

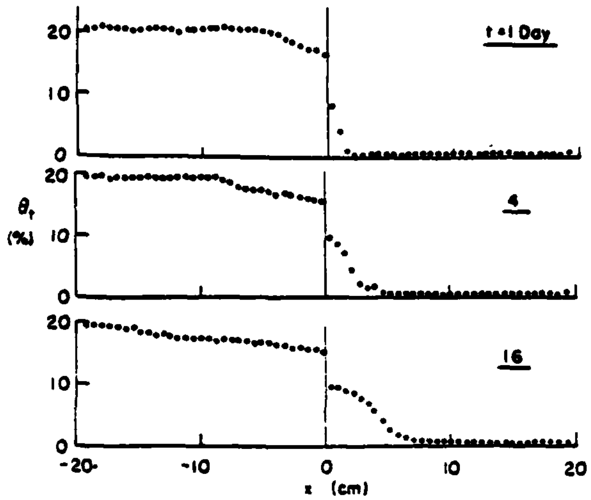

Figure 3. Water movement in Morin clay with ice present. θ_o = 20.0%; T = -1.0°C.

$$\theta = .1269\ (-T)^{-.5080} \qquad (12)$$

Since the tests were run at -1°C, the initial water and ice contents were 12.7% and 7.3% respectively. Some test results are shown in Fig. 3, which indicate that water moved slowly but steadily from the wet part of the soil column to the dry part.

Supplemental tests were carried out to determine if the phase equilibrium of the unfrozen water was maintained. The total water content of the soil was measured gravimetrically while the unfrozen water content was determined by NMR. As the total water content decreased the unfrozen water content remained at the initial value until the total water content dropped to about 14%. The unfrozen water content of the wet column remained constant at the equilibrium unfrozen water content at -1°C and any net water removal from the wet soil column was at the expense of the ice. The ice which melted in the wet column absorbed energy from the surroundings but the system temperature did not vary more than ±.01°C. Thus the ice melted so slowly that latent energy could be transferred with a very minor temperature gradient.

If Darcy's law is valid for water flow with ice then

$$q = -\rho_w D(\theta_t) \frac{\partial \theta_t}{\partial x} \qquad (13)$$

Using Eq. (13) it was possible to calculate the diffusivity using the data for θ_t and q as

$$D(\theta_t) = 49.9\ (\theta_{to} - \theta_t)^{.678} \tag{14}$$

By solving the water content equation with a diffusivity given by Eq. (14), the cumulative water flux was predicted as a function of time and position. Since these values compared quite well to the data, a Darcy type law may govern the flow of film water in icy soils under conditions similar to those of the experiment.

FREEZING ZONE TEMPERATURE AND PRESSURE

Although moisture tensions are known to exist below an advancing freezing front, and are suspected to exist within the freezing zone itself, the cause of these tensions has not been adequately explored. Critical assumptions in frost heave theories are not adequately supported by data on temperatures and pressures that exist within the freezing zone. Tests were designed to provide concurrent measurements of temperature and pressure in the moisture phase of freezing silt soils. Graves sandy silt and Northwest silt were frozen unidirectionally from the upper surface with the lateral surfaces insulated. Selected physical properties for these soils are listed in reference [31].

A special apparatus was designed and fabricated to contain the 300-mm-long soil specimen during freezing and to support the measuring devices for temperature and moisture tension [31]. Top and bottom temperatures were maintained to a tolerance of 0.01°C by separate fluid baths; the upper temperature was adjusted daily to advance the freezing front into the specimen at a generally decreasing rate between 20 and 5 mm/day over a 24-day period. De-aired water was supplied continuously to the bottom of the soil through a porous stone connected to an external 4°C water supply held level with the bottom of the specimen.

Temperatures along the length of the soil column were measured using calibrated thermistors mounted within the wall of the containing vessel in direct contact with the soil. These registered soil temperatures within 0.05°C. Moisture tensions were measured in the unfrozen and frozen portions of the soil column by means of special tensiometers designed at CRREL for temperatures above and below 0°C [32]. A 10% solution of ethylene glycol and water was used within the porous cup which remains unfrozen to a temperature of approximately -4°C and does not diffuse into the soil water. Within experimental error the soil moisture tension is found to be the same for the 10% solution as for water.

Figure 4 is an example of the data obtained for the Graves sandy silt. The Northwest soil, a true silt, developed similar features. Tension was registered continuously by the tensiometers and differed significantly from atmospheric pressure only when the soil temperature was below

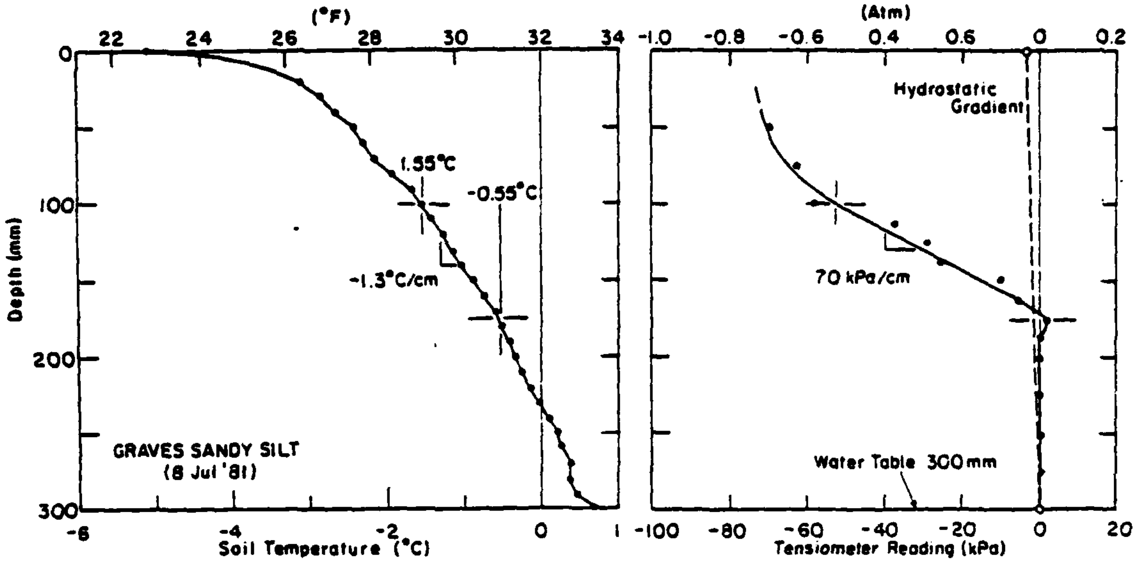

Figure 4. Temperature and apparent moisture tension in freezing zone of Graves sandy silt.

-0.4°C. The value of moisture tension increased rapidly as temperatures became lower, finally reaching an apparent maximum at approximately 70 kPa, after which it remained constant or fell off slightly at a temperature of -1.5°C for the two soils tested. Water flowed through the unfrozen soil at a barely detectable moisture tension gradient, consistent with the measured water intake and the hydraulic conductivity of the soils tested. Moisture tension gradients in the freezing zone reached a maximum of approximately 2 kPa/mm.

Figure 5a is a graph of the moisture tensions and temperatures measured in the freezing zone of the Graves sandy silt plotted for various depths. Figure 5b is a graph of unfrozen water content for this soil, as a function of below-freezing temperature, measured with the NMR procedure.

Combining the data of Fig. 5a and 5b leads to an important new step in the assessment of frost heaving: a moisture-tension characteristic for the unfrozen water behind the freezing front (Fig. 6). The combining of pressure and temperature data assumes that the pressure measured by the tensiometers is that of the unfrozen water itself, i.e. the state of the water is the same in the active freezing zone as it is in the unfrozen water tests. An observed correspondence in the two types of test provides strong support for this assumption: the temperature of nucleation found by ice seeding in the unfrozen water tests (approximately -0.5°C) is the same temperature at which moisture tension first exceeds the hydrostatic level in

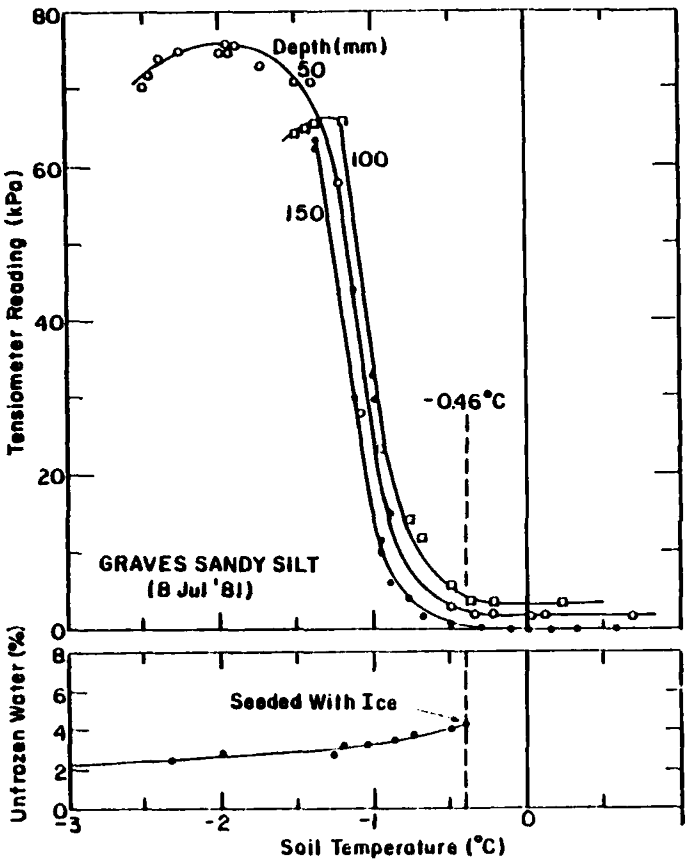

Figure 5. Moisture tension (a) and unfrozen water content (b) versus temperature in freezing zone of Graves sandy silt.

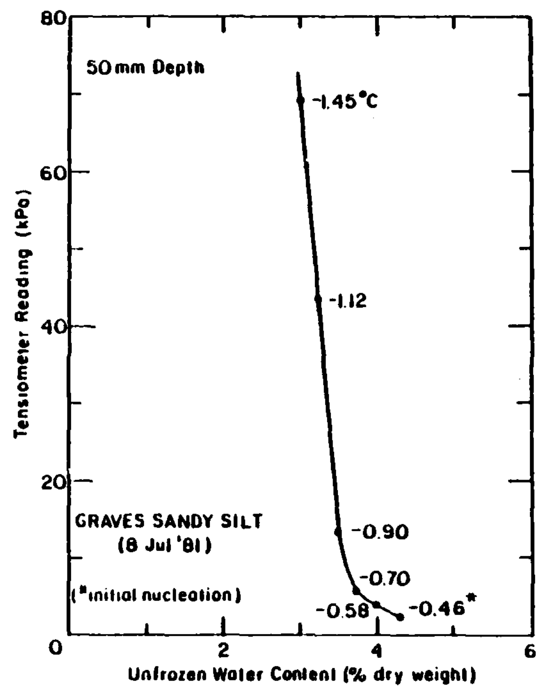

Figure 6. Freezing-zone moisture characteristic; Graves sandy silt.

the advancing freezing front. Moreover, a steep gradient of tension develops at temperatures immediately lower than -0.5°C.

Unlike a moisture characteristic for an unfrozen soil, the freezing-zone moisture characteristic is not independent of temperature; each intersection of moisture tension and unfrozen water content on such a plot corresponds to a narow range of temperatures below 0°C. The range is most likely the result of local variations in gradation, density, or water content in the vicinity of a tensiometer. A normal moisture characteristic corresponds to an air/water interfacial system, while the freezing-zone moisture characteristic apparently conforms to a water/ice interfacial system having a lower surface energy.

CONCLUSIONS

New relations for the flow of water in partially frozen soils are now being incorporated into frost heave models. The thermodynamic equilibrium equations for freezing soil water are well established but the partition of the soil system stresses within freezing zones is poorly understood.

It has been demonstrated that the hydraulic conductivity of water in frozen soils is small but non-zero. Over time spans of several years, which are appropriate for new large scale engineering projects in cold climates, water flow in frozen soils will be significant. The experimental work has shown that Darcy's law is valid for film water flow in ice-free frozen soils and is a reasonable approximation for frozen soil with ice if the ice pressure gradient is negligible.

The capability of deriving the freezing-zone moisture characteristic for a frost-susceptible soil, demonstrated here, is important in the theoretical prediction of the amount of segregated ice in a freezing soil. The empirical correspondence between temperature, moisture tension, and unfrozen water content may lead to physically realistic computer algorithms for the freezing zone. Quantitative predictions from frost heave models will rely on new information of this kind.

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NOMENCLATURE

D diffusivity
D_θ $k/\rho_w g\left(\frac{\partial\psi}{\partial\theta}\right)_T$, moisture diffusivity
D_T $k/\rho_w g\left(\frac{\partial\psi}{\partial T}\right)_\theta$, temperature diffusivity
$\bar{e}_z$ unit vector in z-direction
f free energy
F surface tension function
g gravitational constant
k hydraulic conductivity
L latent heat of fusion
p pressure
q liquid flux
t time
T temperature
T_o freezing temperature of bulk water
v specific volume
x coordinate in flow direction
z coordinate in gravity direction
θ liquid water content
θ_o initial water content
θ_t total water content
ξ $x\,t^{-\frac{1}{2}}$
π osmotic pressure
ρ density
σ surface tension
ψ matric potential
∇ nabla operator

Subscripts

a,i,w air, ice, water, respectively
aw,iw air/water, ice/water respectively